

July-2022

**INTEGRAL EQUATIONS AND CALCULUS OF
VARIATIONS**

Paper-16MAT22C3

[Time allowed : 3 hours] [Maximum marks : 80]

Note : Attempt five questions in all. Questions No. 1 is compulsory.

Section-I

1. (a) From an integral equation for the initial value problem : 8

$$\frac{dy}{dx} - 2\lambda \frac{dy}{dx} - 3y = 0, y(0) = 1, y'(0) = 0$$

- (b) Using successive approximation method, solve

$$u(x) = 1 - \int_0^x (x-t) u(t) dt, \text{ taking } u_0(x) = 1. \quad 8$$

2. (a) Solve the integral equation by using Laplace transform $u(x) = 1 + \int_0^x \sin(x-t) u(t) dt$. Also verify your result. 8

- (b) Find the resolvent Kernel to solve the Volterra integral equation $\mu(x) = f(x) + \lambda \int_0^x e^{x-\xi} u(\xi) d\xi$. 8

Section-II

3. (a) Describe the method of successive approximations to solve Fredholm integral equation 8

- (b) Solve the Fredholm integral equation

$$u(x) = \cos x + \lambda \int_0^x \sin(x-t) u(t) dt \text{ by the method of separable kernel.} \quad 8$$

4. (a) Find the resolvent kernel for the Fredholm integral equation <https://www.mdustudy.com>

$$u(x) = 1 + \lambda \int_0^1 (1 - 3xt) u(t) dt \text{ using Neumann series expansion.} \quad 8$$

- (b) Solve the integral equation

$$u(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) u(\xi) d\xi \text{ by the method of iteration. Also find the condition on } \lambda \text{ for which solution exists.} \quad 8$$

Section-III

5. (a) Construct the Green's function for the BVP $u''(x) + u(x) = f(x) \text{ in } 0 < x < 1, u(0) = 0, u(1) = 1$ by using its properties. 8

- (b) Reduce the BVP $u''(x) + \lambda u(x) = x$, $0 < x < \frac{\pi}{2}$

$u(0)=0, u\left(\frac{\pi}{2}\right)=0$ to a Fredholm integral equation.
using the Green's function method.

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6. Define the Green's function of BVP :

$$\frac{d}{dx} \left[r(x) \frac{du}{dx} \right] + [q(x) + \lambda \rho(x)] u(x) = f(x) \text{ in } a < x < b,$$

$$\alpha_1 u(a) + \alpha_2 u'(a) = 0$$

$$\beta_1 u(b) + \beta_2 u'(b) = 0$$

and construct $G(x, \xi)$ by using the basic four properties.

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Section-IV

7. (a) Define the variation of a functional. Show that a necessary condition for a differentiable functional to have an extremum for $y = \hat{y}$ is that its variation vanishes for $y = \hat{y}$.

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- (b) Among all the curves joining two given points (x_0, y_0) and (x_1, y_1) , find the one which generates the surface of minimum area when rotated about the x -axis.

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8. (a) Solve the variational problem

$$J[y] = \int_a^b \left\{ y^2 + \frac{(y')^2}{x} \right\} dx, \quad y(1)=0, y(2)=1 \quad 8$$

- (b) Find the geodesics of the circular cylinder
 $\vec{r} = (a \cos \phi, a \sin \phi, z)$

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Section-V

9. (a) Define Volterra integral equation of second kind and give an example :

- (b) Define Leibnitz's rule for differentiation under integral sign. <https://www.mdustudy.com>

- (c) What do you mean by non-homogenous integral equation of Fredholm type ?

- (d) What is Fredholm integral equation of first kind and give an example ?

- (e) State Hilbert-Schmidt theorem for symmetric kernels.

- (f) Define a self-adjoint operator.

- (g) What is Brachistochrone problem ? Give an example.

- (h) What is problem of Geodesics ? Give an