7. (a) Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$. 7.5

(b) Find the directional derivative of the $f = xy^2 + yz^2$ at the point $(2, -1, 1)$ in direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 7.5

UNIT – IV

8. (a) Find the Inverse of Matrix : 7.5

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

using Elementary transformations.

(b) Find the values of a and b for which the equations $x + ay + z = 3$, $x + 2y + 2z = b$ $x + 5y + 3z = 9$ are consistent. When will these equations have unique solution ? 7.5

9. (a) Find the Eigen values and Eigen vectors of Matrix : 7.5

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem and find the Inverse of matrix : 7.5

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

———————

3007-3,650 -(P-4)(Q-9)(18)   (4)

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit. Question No. **1** is *compulsory*. All questions carry equal marks.

1. (a) Determine Rank of Matrix. : $6 \times 2.5 = 15$

$$A = \begin{bmatrix} 2 & 5 & 6 \\ 1 & 2 & 3 \\ 3 & 7 & 9 \end{bmatrix}$$

(b) Evaluate the following limit

$$f(x, y) = \underset{x \to 1}{Lt} \frac{2x^2 y}{x^2 + y^2 + 1}$$

3007-3,650 -(P-4)(Q-9)(18)   **P. T. O.**

(c) If $z = \sin^{-1}\left[\dfrac{x^2 + y^2}{x - y}\right]^{y \to 2}$ Determine $\left(\dfrac{\partial z}{\partial y}\right)$.

(d) State the Result of Lagranges's mean value theorem.

(e) Compute $B\left(\dfrac{5}{2}, \dfrac{3}{2}\right)$.

(f) Evaluate Curl $\left[e^{xyz}(i + j + k)\right]$.

UNIT – I

2. (a) Evaluate :   8

  (i) $\underset{x \to 1}{Limit}\ \dfrac{x^x - x}{x - 1 - \log x}$

  (ii) $\underset{x \to \frac{\pi}{2}}{Lt}\ (\tan x)^{\cos x}$

(b) Show that the evolute of the cycloid $x = a(\theta - \sin\theta),\ y = a(1 - \cos\theta)$ is another equal cycloid.   7

3. (a) Find the volume of the solid generated by revolving the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ about y-axis.   7

(b) Prove that :

  (i) $\displaystyle\int_0^1 \dfrac{x\,dx}{\sqrt{1 - x^5}} = \dfrac{1}{5}B\left(\dfrac{2}{5}, \dfrac{1}{2}\right)$ 8

  (ii) $\displaystyle\int_0^1 \dfrac{dx}{\sqrt{1 + x^4}} = \dfrac{1}{4\sqrt{2}}B\left(\dfrac{1}{4}, \dfrac{1}{2}\right)$

UNIT – II

4. (a) Test the convergence of following series :   7.5

$$1 + \dfrac{x}{2} + \dfrac{x^2}{5} + \dfrac{x^3}{10} + \ldots + \dfrac{x^n}{n^2 + 1} + \ldots.$$

(b) Test for convergence the series :   7.5

$$\sum \dfrac{4.7\ldots\ldots(3n + 1)x^n}{1.2.3.4.\ \ldots\ldots n}$$

5. (a) Test the convergence of an Alternating series : 7.5

$$\dfrac{1}{1.2} - \dfrac{1}{3.4} + \dfrac{1}{5.6} - \dfrac{1}{7.8} + \ldots\ldots$$

(b) Find a Fourier series of $f(x) = x^2$ over the interval. $[-\pi,\ \pi]$.   7.5

UNIT – III

6. (a) If $v = (x^2 + y^2 + z^2)^{-1/2}$ then prove that

$$\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 v}{\partial z^2} = 0 .$$ 7.5

(b) If $z = f(x, y)$ and $x = e^u + e^{-v},\ y = e^{-u} - e^v$ prove that $\dfrac{\partial z}{\partial u} - \dfrac{\partial z}{\partial v} = x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y}$.   7.5