Roll No. ....

24022

B. Tech. 4th Semester (Re-Appear)
(Common for all Branches)
Examination – October, 2020
MATHS-III
Paper : Math-201-F

Time : 1.45 hours]   [Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt any *three* questions. All questions carry equal marks.

1. (a) Explain $f(x) = \begin{cases} 0 & \text{for} \quad 0 \le x \le \pi \\ 1 & \text{for} \quad x > \pi \end{cases}$ as Fourier sine integral.

   (b) If $\tan \log (x + iy) = a + ib$ and $a^2 + b^2 \ne 1$, then prove that :
   $$\tan \log(x^2 + y^2) = \frac{2a}{1 - (a^2 + b^2)}$$

   (c) Find the residue of $\operatorname{cosec}^2 z$ at $z = 0$.

   (d) Samples of size 10 & 14 were taken from two normal populations with S. D. 3.5 and 5.2. The samples means were found to be 2.3 and 18.6. Test whether means of two populations are the same at 5% level.

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2. (a) Expand $f(x) = e^{-x}$ as Fourier series in interval $(-l, l)$.

   (b) State and prove convolution theorem for Fourier transformation.

3. (a) Obtain Fourier series for :
   $$f(x) = \begin{cases} \pi x & \text{for} \quad 0 \le x \le 1 \\ \pi(2 - x) & \text{for} \quad 1 \le x \le 2 \end{cases}$$

   (b) Find Fourier transform of :
   $$f(x) = \begin{cases} 1 - |x| & \text{for} \quad |x| < 1 \\ 0 & \text{for} \quad |x| > 1 \end{cases}$$

4. (a) Find the values of A and B such that the function $f(z) = x^2 + Ay^2 - 2xy + i(Bx^2 - y^2 - 2xy)$ is analytic. Also find $f'(z)$.

   (b) Verify Cauchy's integral theorem by integrating e^{iz} along the boundary of the triangle with vertices at the point $1 + i$, $-1 + i$ and $-1 - i$.

5. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at origin, even though C-R equations are satisfied there at.

   (b) State and prove Cauchy's integral theorem.

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6. (a) Evaluate :

$$\int_0^\pi \frac{d\theta}{a + b\cos\theta}$$

where $a > |b|$.

(b) Evaluate :

$$\int_0^\infty \frac{\cos ax}{x^2 + 1} dx \ (a \geq 0)$$

7. (a) The bags A, B, C contains 4 red, 3 black, 2 white; 3 red, 4 black, 4 white and 5 red, 2 black, 6 white balls respectively. If a bag is selected at random and a ball is drawn from it, find the probability that the ball drawn is red.

(b) Data was collected over a period of 10 years, showing number of deaths from horse kicks in each of the 200 army corps. The distribution of deaths was as follows :

No. of deaths	:	0	1	2	3	4	Total
Frequency	:	109	65	22	3	1	200

Fit a Poisson distribution to the data and calculate the theoretical frequencies.

. (a) A random sample of 200 villages from Coimbatore district gives the mean population per village at 485 with a S. D. of 50. Another random sample of the same size from the same district gives the mean population per village at 510 with S. D. of 40. Is the difference between the mean values given by the two samples statistically significant ? Justify your answer.

(b) A sample of 20 items has mean 42 units and S. D. 5 units. Test the hypothesis that it is a random sample from a normal population with mean 45 units.

9. Using dual simplex method :

Minimize : $Z = x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

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