24022

[Graph Paper]

B.Tech. 4th Semester F-Scheme

(Common for All Branches) Examination,

May-2019

MATHEMATICS-III

Paper-Math-201-F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt **five** questions in total selecting **one** question from each section. **Question No. 1 is compulsory.***

1. (a) Find the value of a_n in the Fourier series of $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$,

   (b) Separate into real and imaginary parts Log (4+3i).

   (c) Define residue. Write statement of Cauchy's residue theorem.

   (d) Solve the following LPP graphically :

   Minimize $Z = 3x + 2y$

   subject to the constraints

   $5x + y \geq 10$, $x + y \geq 6$, $x + 4y \geq 12$, $x, y \geq 0$.

Section-A

2. (a) Find the Fourier serial expansion for :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + --- = \frac{\pi^2}{8}$.

(b) Express $f(x) = x$ as a half – range sine series in $0 < x < 2$.

3. Solve the partial differential equation :

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \quad u > 0, t > 0$$

subject to the conditions :

(a) $u(0,t) = 0, t > 0$

(b) $u(x, 0) = e^{-x}, x > 0$

(c) u and $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$

Section-B

4. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the orign, even though C–R equations are satisfied there at.

   (b) Determine the analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$.

5. (a) Evaluate $\oint_c |z|^2 dz$, around the square with vertices at (0,0), (1,0), (1,1) and (0,1).

(b) Use Cauchy's integral formula to evaluate $\oint_C \frac{2^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 2$.

Section-C

6. (a) Expand $\frac{1}{z^2 - 3z + 2}$ in the region :

(i) $1 < |z| < 2$

(ii) $|z| < 2$

(b) Evaluate $\oint_C \frac{z^3}{(z-1)^4 (z-2)(z-3)} dz$

Where C is the circle $(z) = \frac{5}{2}$. By using residue theorem.

7. (a) A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and the variance of the number of successes.

(b) Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six ?

Section-D

8. (a) The sales in a super market during a week are given below. Test the hypothesis that the sales do not depend on the day of the week at 5% level of significance :

Days	:	Mon	Tues	Wed	Thus	Fri	Sat
Sales (in 1000 Rs.)	:	65	54	60	56	71	84

(b) A sample of 18 items has a mean 24 units and standard deviation 3 units. Test the hypothesis that it is a random sample from a normal population with mean 27 units.

9. Using dual simplex method

Maximize $z = -3x_1 - 2x_2$

Subject to $x_1 + x_2 \geq 1$,

$x_1 + x_2 \leq 7$,

$x_1 + 2x_2 \geq 10$,

$x_2 \leq 3$,

$x_1, x_2 \geq 0$