

## Section-D

8. (a) Form the partial differential equation by eliminating the function  $f$  from the relation

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$$

- (b) Solve:

$$\frac{\partial^2 z}{\partial x^2} = a^2 z \text{ given that, when } x=0,$$

$$\frac{\partial z}{\partial x} = a \sin y \text{ and } \frac{\partial z}{\partial y} = 0.$$

- (c) Solve:

$$x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2).$$

9. A tightly stretched string with fixed end points  $x=0$  and  $x=l$  is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity  $\lambda x(l-x)$ , find the displacement of the string at any distance  $x$  from one end at any time  $t$ .

## B. Tech 2nd Semester Examination,

May-2016

MATHEMATICS-II

Paper-Math-102 F

Common for all branches

Time allowed : 3 hours]

[Maximum marks : 100

*Note : Attempt five questions in total selecting at least one from each section. Question No. 1 is compulsory.*

1. (a) Define Geometrical interpretation of gradient.  
 (b) State Green's theorem in the plane.  
 (c) Solve  $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$ .  
 (d) Find the P. I. of  $(D^2 - 4)y = e^{2x}$ .  
 (e) Find the Laplace transform of  $\cosh at \sin at$ .  
 (f) Find the Laplace transform of  $\frac{\sin^2 t}{t}$ .  
 (g) Solve  $p^2 + q = q^2$ .  
 (h) Solve:

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$$

by method of separation of variables.

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## Section-A

2. (a) Find the directional derivative of the function  $f = x^2 - y^2 + 2z^2$  at the point P (1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4).

- (b) A vector field is given by

$$\vec{A} = (x^2 + xy^2) \hat{i} + (y^2 + x^2y) \hat{j}.$$

Show that the field is irrotational and find the scalar potential.

3. Verify Green's theorem in the plane for

$$\oint_C (2xy - x^2) dx + (x^2 + y^2) dy,$$

where C is the boundary of the region enclosed by  $y = x^2$  and  $y^2 = x$ .

## Section-B

4. (a) State and prove the necessary and sufficient condition for the differential equation  $M dx + N dy = 0$  to be exact.
- (b) Find the orthogonal trajectory of the family of curves.  $r = a(1 + \cos \theta)$ .

5. (a) Solve:

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

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- (b) An uncharged condenser of capacity C is charged by applying an e.m.f.

$E \sin \frac{t}{\sqrt{LC}}$ , through leads of self-inductance L and negligible resistance. Find the charge on one of the plates at time t.

## Section-C

6. (a) Find Laplace transform of the function f(t) defined as  $f(t) = |t-1| + |t+1| + |t+2| + |t-2|$ ,  $t \geq 0$ .

- (b) Find the inverse Laplace transform of

$$\log \frac{s^2 + 1}{(s-1)^2}.$$

- (c) Find the inverse Laplace transform of

$$\frac{1}{(s^2 + 1)(s^2 + 9)}$$

by convolution theorem.

7. (a) Solve the integral equation

$$\int_0^t \frac{y(u)}{\sqrt{t-u}} du = 1 + t + t^2$$

by Laplace transform method.

- (b) Find the Laplace transform of the triangular wave function of period 2c given by

$$f(t) = t, 0 < t < c$$

$$= 2c - t, c < t < 2c.$$

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[P.T.O.]