

3007

B. Tech. 1st Semester (ME)

Examination – December, 2018

MATHEMATICS - I (CALCULUS & MATRICES)

Paper : BSC-Math-101-G

Time : Three Hours]

[Maximum Marks : 75

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit. Question No. **1** is *compulsory*. All questions carry equal marks.

1. (a) Determine Rank of Matrix. : $6 \times 2.5 = 15$

$$A = \begin{bmatrix} 2 & 5 & 6 \\ 1 & 2 & 3 \\ 3 & 7 & 9 \end{bmatrix}$$

(b) Evaluate the following limit

$$f(x, y) = \lim_{x \rightarrow 1} \frac{2x^2y}{x^2 + y^2 + 1}$$

(c) If $z = \sin^{-1} \left[\frac{x^2 + y^2}{x - y} \right]^{y \rightarrow 2}$ Determine $\left(\frac{\partial z}{\partial y} \right)$.

(d) State the Result of Lagranges's mean value theorem.

(e) Compute $B \left(\frac{5}{2}, \frac{3}{2} \right)$.

(f) Evaluate $\text{Curl} [e^{xyz}(i + j + k)]$.

UNIT – I

2. (a) Evaluate : 8

(i) $\lim_{x \rightarrow 1} \frac{x^x - x}{x - 1 - \log x}$

(ii) $\lim_{x \rightarrow \frac{\pi}{2}} (\tan x)^{\cos x}$

(b) Show that the evolute of the cycloid $x = a(\theta - \sin \theta), y = a(1 - \cos \theta)$ is another equal cycloid. 7

3. (a) Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about y-axis. 7

(b) Prove that :

(i) $\int_0^1 \frac{x dx}{\sqrt{1-x^5}} = \frac{1}{5} B \left(\frac{2}{5}, \frac{1}{2} \right)$ 8

$$(ii) \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}} B\left(\frac{1}{4}, \frac{1}{2}\right)$$

UNIT - II

4. (a) Test the convergence of following series : 7.5

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots + \frac{x^n}{n^2+1} + \dots$$

- (b) Test for convergence the series : 7.5

$$\sum \frac{4.7. \dots (3n+1)x^n}{1.2.3.4. \dots n}$$

5. (a) Test the convergence of an Alternating series : 7.5

$$\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \dots$$

- (b) Find a Fourier series of $f(x) = x^2$ over the interval. $[-\pi, \pi]$ 7.5

UNIT - III

6. (a) If $v = (x^2 + y^2 + z^2)^{-1/2}$ then prove that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0. \quad 7.5$$

- (b) If $z = f(x, y)$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$. 7.5

7. (a) Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$. 7.5
 (b) Find the directional derivative of the $f = xy^2 + yz^2$ at the point (2, -1, 1) in direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 7.5

UNIT - IV

8. (a) Find the Inverse of Matrix : 7.5

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$$

using Elementary transformations.

- (b) Find the values of a and b for which the equations $x + ay + z = 3$, $x + 2y + 2z = b$, $x + 5y + 3z = 9$ are consistent. When will these equations have unique solution? 7.5

9. (a) Find the Eigen values and Eigen vectors of Matrix : <http://www.HaryanaPapers.com> 7.5

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

- (b) Verify Cayley-Hamilton theorem and find the Inverse of matrix : 7.5

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$