Roll No. ....

3008

B. Tech. 1st Semester (CSE) Examination – March, 2021

MATH - I (Calculus and Linear Algebra)

Paper : BSC-MATH-103-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is *compulsory*. All questions carry equal marks.

1. Answer the following questions in brief : 2.5 × 6 = 15

(a) State Taylor's and Maclaurin theorem with remainders.

(b) Examine the linear independence of the following set of vectors

$$\{(1, 2, 3), (1, 1, 1), (0, 1, 2)\}$$

(c) Show that for two matrices A and B, $(AB)^{-1} = B^{-1}A^{-1}$.

(d) Show that the function $T : R^3 \to R^2$ defined by $T(x, y, z) = (|x|, y - z)$ is not a linear transformation.

(e) If $T : U \to V$ is a linear transformation, then show that $\ker T$ is a subspace of U.

(f) If A is a square matrix, prove that $(A + A')$ is symmetric and $(A - A')$ is skew-symmetric.

UNIT – I

2. (a) Evaluate $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$ 7

(b) Prove that equation of the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$ 8

3. (a) Find the volume generated by revolution about initial line of $r = a(1 - \cos\theta)$. 7

(b) Prove that : 8

(i) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}} = \frac{1}{5}\beta\left(\frac{2}{5}, \frac{1}{2}\right)$

(ii) $\int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{1}{4\sqrt{2}}\beta\left(\frac{1}{4}, \frac{1}{2}\right)$

UNIT – II

4. (a) If $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$, compute AB and BA and show that $AB \neq BA$. 7½

(b) Find the rank of a matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ 7½

5. (a) Using Cromer's rule, solve the following equation :

$x + 3y + 6z = 2$; $3x - y + 4z = 9$; $x - 4y + 2z = 7$. 7½

(b) Solve the following system of equations by using Gauss-Jordan elimination method : 7½

$4y + z = 2$; $2x + 6y - 2z = 3$; $4x + 8y - 5z = 4$.

UNIT – III

6. (a) Show that the set {(2, 1, 4), (1, –1, 2), (3, 1, –2)} form a basis of R^3. 7

(b) If $T : R^4 \rightarrow R^3$ is a linear transformation defined by $T(1, 0, 0, 0) = (1, 1, 1)$, $T(0, 1, 0, 0) = (1, -1, 1)$, $T(0, 0, 1, 0) = (1, 0, 0)$ and $T(0, 0, 0, 1) = (1, 0, 1)$, then verify that Rank T + Nullity T = dim R^4. 8

7. (a) Let $T : U \rightarrow V$ be invertible linear transformation and $T^{-1} : V \rightarrow U$ be its inverse. Then show that T^{-1} is also a linear transformation. 7½

(b) If T_1 and T_2 be two linear operators defined on R^2 s.t. $T_1(x, y) = (x + y, 0)$ and $T_2(x, y) = (-y, x)$. Find a formula for the operators : 7½

(i) T_1T_2

(ii) T_2T_1

(iii) T_1^2

UNIT – IV

8. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ 8

(b) Find the values of a, b, c if $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal. 7

9. (a) Diagonalise the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$. 7½

(b) Using Gram-Schmidt orthogonalization process, construct an orthonormal basis of $V_3(R)$ with standard inner product defined on it, given the basis $u_1 = (1, 1, 1)$, $u_2 = (1, -2, 1)$ and $u_3 = (1, 2, 3)$. 7½