94059

B. Sc. (Hons.) Mathematics
5th Semester Old/New Scheme
Examination – February, 2022

REAL ANALYSIS

Paper : BHM-351

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section-V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) If a function f is defined on $[0, a]$, $a > 0$ by $f(x) = x^3$, then show that f is Riemann integrable on $[0, a]$ and $\int_0^a f dx = \frac{a^4}{4}$.

(b) Prove by summation $\int_a^b \frac{1}{\sqrt{x}} dx = 2(\sqrt{b} - \sqrt{a})$.

2. (a) If 'f' is integrable on $[a, b]$ and F is the primitive of f on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$.

(b) Prove the inequality :

$$\frac{\sqrt{3}}{8} \le \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx \le \frac{\sqrt{2}}{6}$$

SECTION – II

3. (a) Examine the convergence of $\int_0^1 x^{n-1} \log x dx$.

(b) Discuss the convergence of Gamma function.

4. (a) Examine the convergence of :

$$\int_0^\infty \frac{\cos x}{\sqrt{x^2 + x}} dx$$

(b) Evaluate $\int_0^{\pi/2} \log(\alpha^2 \cos^2 \theta + \beta^2 \sin^2 \theta) d\theta$.

SECTION – III

5. (a) Prove that if (X, d) is a metric space and x, y, z are points of X, then $d(x, y) \ge |d(x, z) - d(z, y)|$.

(b) Let A be a subset of metric space (x, d). Then A° is the union of all open sets contained in A.

6. (a) A subset F of a metric space (X, d) is closed iff F contains all its limit point i.e. $d(F) \leq F$.

(b) A subspace Y of a complete metric space X is complete iff it is closed.

SECTION – IV

7. (a) Let (X, d) and (Y, d^*) be two metric spaces and let f, g be two continuous functions of X into Y. Then the set $\{x \in X : f(x) = g(x)\}$ is a closed subset of X.

(b) Every compact metric space is complete.

8. (a) Prove that continuous image of a compact metric space is compact.

(b) Let (X, d) be a metric space and let $\{C_\lambda\}_{\lambda \in \wedge}$ be a nonempty collection of connected subsets of X such that $\bigcap_{\lambda \in \wedge} C_\lambda \neq \phi$. Then $\bigcup_{\lambda \in \wedge}\{C_\lambda\}$ is a connected.

SECTION – V

(Compulsory Question)

9. (a) Compute $\int_0^3 [x]\,dx$, where $[x]$ is the greatest integer function.

(b) Discuss the convergence and show that $\int_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}(bx)}{x} = \frac{\pi}{2}\log\left|\frac{a}{b}\right|$.

(c) Define metric and metric space.

(d) If d be the usual metric $d(x,y) = |x - y|$ for $x, y \in [0, 1]$, describe $S\left(0, \frac{1}{8}\right)$.

(e) Show that in a discrete metric space (X, d), every subset of X is open.

(f) State finite intersection property in a metric space.