

Roll No.

41251

**B. Sc. (Hons.) Maths 4th Semester
Examination – May, 2019**

SEQUENCES AND SERIES

Paper : BHM241

Time : Three hours] [Maximum Marks : 60

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 9 (Unit - V) is *compulsory*. All questions carry equal marks.

UNIT – I

1. (a) Prove that between two distinct real numbers, there are infinitely many irrational numbers.
- (b) Prove that A° of a set A is the largest open subset of A .
2. (a) Prove that infimum of a set A are also infimum of \bar{A} and contained in \bar{A} as A is bounded below.

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- (b) State and prove converse of Heine Borel Theorem.

UNIT – II

3. (a) By definition, show that the sequence

$$\left\langle \frac{h^2 + 3h + 5}{2n^2 + 5n + 7} \right\rangle \text{ converges to } \frac{1}{2}.$$

- (b) Let $\langle a_n \rangle$ be a sequence such that $a_n \neq 0 \forall n \in \mathbb{N}$ and $\frac{a_n + 1}{a_n} \rightarrow \rho$. If $|\rho| < 1$, then $\lim_{n \rightarrow \infty} a_n = 0$.

4. (a) Show that the sequence $\langle a_n \rangle$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2^{n-1}}$$
 does not converge.

- (b) Discuss the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}, x > 0$

(ii) $\sum_{n=1}^{\infty} \frac{x^n}{a^n + x^x}$

UNIT – III

5. Test for convergence of the following series :

(2)

- (a) $\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \dots$
- (b) $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$
- (c) $\frac{a}{b} + \frac{a(a+1)}{b(b+1)} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \dots$

6. (a) State and prove Gauss Test for the convergence of an infinite series.

(b) Discuss the convergence of the following series

(i) $\sum_{n=1}^{\infty} \left(n^{1+\frac{1}{n}} \right)^{-n}$

(ii) $\frac{a+x}{1!} + \frac{(a+2x)^2}{2!} + \frac{(a+3x)^3}{3!} + \dots$

UNIT - IV

7. (a) Test the convergence and absolute convergence of the following series :

(i) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{(n+1)!}$

(ii) $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\sqrt{n^2 - 1} - n \right)$

(b) Prove that the infinite product $\prod_{n=1}^{\infty} \cos \frac{x}{2^n}$ converges to $\frac{\sin x}{x}$, where x is an arbitrary fixed non-zero number.

(a) Test the convergence or divergence of the series :

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right) \sin(n\theta + \alpha)$$

(b) State and prove Mertin's theorem.

UNIT - V

(a) Give an example to show that :

$$A^{\circ} \cup B^{\circ} \neq (A \cup B)^{\circ}.$$

(b) Find the derived set of $\left\{ 1 + \frac{1}{2^n} \mid n \in \mathbb{N} \right\}$.

(c) Discuss the boundedness of the sequence :

$$\left\langle \frac{2n+3}{3n+4} \right\rangle.$$

(d) State Logarithmic Test.

State Abel's test in arbitrary series.

Discuss the convergence of the infinite product :

$$\prod_{n=2}^{\infty} \left(1 + \frac{(-1)^n}{\sqrt{n}} \right).$$