Roll No. : .....

Total No. of Questions : **9**] [Total No. of Pages : **7**

97663

B.C.A. 1st Semester (New)
(Full & Reappear)
Examination, March-2021

MATHEMATICS

Paper-BCA-103

*Time : **Three** Hours]* *[Maximum Marks : **80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note :– Attempt *five* questions in all, selecting *one* question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

1. (a) Compute 3A + 4B if :

$$A = \begin{bmatrix} 2 & 3 \\ -1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}$$

(b) Write the power set of {2, 5, 10}.

(c) Evaluate :

$$\lim_{x \to 0} \frac{1-\cos 2x}{x^2}$$

(d) Find the domain of :

$$y = \sqrt{x-5}$$

(e) Find $\frac{dy}{dx}$, when :

$$y = \frac{3x^2+1}{x}$$

(f) Find $\frac{dy}{dx}$, when :

$$y = a^{7x+4}.$$

(g) Evaluate :

$$\int \sqrt{x}(x^2+2x+3)dx$$

(h) Evaluate :

$$\int \frac{dx}{1-\cos x}$$

2×8=16

Unit–I

2. (a) In a group of 400 people, 250 can speak English only and 70 can speak Hindi only. Find :

(i) How many can speak both English and Hindi ?

(ii) How many can speak English ?

(iii) How many can speak Hindi ? 8

(b) If :

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

show that : $A^2 - 7A + 10I = 0$ where I is a unit matrix. 8

3. (a) Prove that :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$$

8

(b) Solve the following system of equations using matrices :

$$2x + 8y + 5z = 5$$
$$x + y + z = -2$$
$$x + 2y - z = 2$$ 8

Unit–II

4. (a) In the set N of all natural numbers, let a relation R be defined by :

$$R = \{(x, y) : x \in N, y \in N, x - y \text{ is divisible by } 5\}$$

Prove that R is an equivalence relation. 8

(b) (i) If :

$$f(x) = x^5 - \frac{1}{x^5},$$

find the value of $f(x) + f\left(\frac{1}{x}\right)$. 4

(ii) If :

$$f(x) = x + \frac{1}{x},$$

prove that :

$$[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$$ 4

5. (a) Evaluate :

$$\lim_{x \to 0}\left(\frac{\operatorname{cosec} x - \cot x}{x}\right)$$ 8

(b) Discuss the continuity of the function $f(x)$ at $x = \frac{1}{2}$:

$$f(x) = \begin{cases} \frac{1}{2} - x, & \text{if } 0 \le x < \frac{1}{2} \\ 1, & \text{if } x = \frac{1}{2} \\ \frac{3}{2} - x & \text{if } \frac{1}{2} < x \le 1 \end{cases}$$ 8

Unit–III

6. Differentiate the following functions w.r.t. x :

(i) $y = \frac{x}{\sin x}$

(ii) $y = \sqrt{\frac{1-x}{1+x}}$

(iii) $y = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$

(iv) $y = \tan^{-1}\left(\sqrt{1+x^2}+x\right)$ 4×4=16

7. (a) Find $\frac{dy}{dx}$, if :

$y = x^{\sin x} + (\sin x)^x$. 8

(b) Find $\frac{dy}{dx}$, when :

$x = a(1 + \cos \theta)$,

$y = a(\theta + \sin \theta)$ 8

Unit–IV

8. Evaluate the following integral :

(a) $\int \tan^{-1}(\sec x + \tan x)dx$ 8

(b) $\int \frac{x^2 \tan^{-1} x}{1+x^2} dx$ 8

9. Evaluate the following integral :

(a) $\int \frac{x\,dx}{(x+2)(3-2x)}$ 8

(b) $\int_0^{\pi} \log(1+\cos x)dx$ 8