

**97663**

**B.C.A. 1st Semester (New)**  
**Examination-December, 2013**  
**Mathematics**  
**Paper-BCA-103**

**Time : 3 hours**

**Max. Marks : 80**

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Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

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**Note :** Attempt **five** questions in all, selecting **one** question from each unit. Question no. **1** is **compulsory**.

1. (a) Define union and Intersection of set with examples.  $2 \times 8 = 16$
- (b) Write the set in Roster form  
 $A = \{x : |x - 3| < 8, x \text{ is an integer}\}$
- (c) Construct  $2 \times 2$  Matrix  $A = [a_{ij}]$ , such that  $a_{ij} = \frac{(i + 2j)^2}{2}$
- (d) Define equivalence relation with example

(e) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{e^x - e^{-x}}{x} \right)$

(f) if  $y = \frac{1 - \sqrt{x}}{1 + \sqrt{x}}$ , find  $\frac{dy}{dx}$

(g)  $\int \frac{x}{1 + \sqrt{x}} dx$

(h)  $\int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx$

### Unit-I

2. (a) Prove  $(A \cup B)' = A' \cap B'$  8

(b) If  $A = \{1, 2, 3\}$ ,  $B = \{2, 3, 4\}$ ,  $C = \{1, 3, 4\}$ ,  
 $D = \{2, 4, 5\}$ , verify that 8

$$(A \times B) \cap (C \times D) = (A \cap C) \cap (B \cap D)$$

3. (a) Prove that 8

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-z)(y-z)(z-x)$$

(b) If  $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ , verify that

$$A \cdot (\text{adj} \cdot A) = (\text{adj} \cdot A) \cdot A = (A)I_3 \quad 8$$

## Unit-II

4. (a) Show that the relation  $R$  in the set  $R$  of real Numbers defined as  $R = \{(a, b) : a \leq b\}$ , is reflexive and Transition but not symmetric. 8

(b) If  $f(x) = \log \frac{1+x}{1-x}$ , show that  $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$  8

5. (a) Evaluate :- 8

(i)  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$

(ii)  $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 6x}$

(b) Show that  $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$  does not exist 8

## Unit-III

6. (a) If  $y = \frac{x}{x+4}$ , show that  $\frac{xdy}{dx} = y(1-y)$  8

(b)  $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$  d.w.r.to  $x$  8

7. (a) If  $y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ , find  $\frac{dy}{dx}$  8

(b) If  $y = (x + \sqrt{x^2 + a^2})^4$ , then Prove that

$$\frac{dy}{dx} = \frac{xy}{\sqrt{x^2 + a^2}} \quad 8$$

#### Unit-IV

8. (a) Evaluate 8

(i)  $\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$

(ii)  $\int e^x \cos x dx$

(b) (i)  $\int_1^2 \frac{1}{x(1+\log x)} dx$  8

(ii)  $\int_0^{\pi/2} \sqrt{1+\sin x} dx$

9. (a) Evaluate 8

(i)  $\int \frac{xe^x}{(1+x)^2} dx$

(ii)  $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

(b) (i)  $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}} dx$

(ii)  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x + \sqrt{\sin x}}} dx$  8